Journal of Applied Mechanics and Technical Physics, Vol. 45, No. 6, pp. 828–833, 2004

ELECTROMAGNETIC SOUNDING OF OIL AND GAS FORMATIONS

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UDC 532.546:550.820.7

A mathematical model is proposed that describes electrical conductivity variation in the near-well zone during drilling formations containing three immiscible phases: oil, gas, and a small amount of native salt water. It is assumed that borehole drilling is performed using a clay–water solution, the mass-exchange process between the moving mud filtrate and immovable native water is infinitely fast, and displacement of the gas phase occurs by piston flow. The redistribution of the immiscible phases is described by the conventional Buckley–Leverett equations. The electromagnetic response of the medium is interpreted using the earlier proposed method of probabilistic convolutions.

Key words: *three-phase filtration, mass exchange, electrical conductivity, mud filtrate, electromagnetic sounding.*

Introduction. A radically new method of interpreting electromagnetic sounding data from boreholes penetrating aquifers and oil and gas formations is proposed in [1]. The method is based on mathematical modeling of the processes involved in drilling mud filtrate invasion: immiscible filtration of fluids, instantaneous salt exchange between the moving water filtrate and connate native salt water; use is made of the focusing properties of devices of high-frequency isoparametric induction logging (HFIIL) or high-frequency electromagnetic logging (HFEL).

The representation of the induction resistivities $\bar{R}_i$ in the form of integral convolutions

$$\bar{R}_i = \int\limits_0^\infty R(r)\rho_i(r)r\,dr \qquad (1)$$

of the actual electric resistivities $R(r)$ of annular segments of the near-well zone with the spatial distribution density of the detector sensitivities

$$\rho_i = \frac{1}{2\sqrt{2\pi}\sigma x_i}\exp\Big(-\frac{\sigma^2}{2}\Big)\exp\Big(-\frac{1}{2\sigma^2}\ln^2\frac{x}{x_i}\Big)$$

gives the functional relationship between the resistivities $\bar{R}_i$ and the initial physical characteristics of the formations. Here $x_i = r_i^2$, $x = r^2$ (r_i are the centers of sensitivities), σ is the dispersion of the device, and r is the radial coordinate. More details of the method and examples of its use can be found in a paper [2], which gives typical sounding curves for three cases of invasion: into aquifers, oil formations, and gas formations.

In the case of filtrate invasion into aquifers, the distribution of the actual resistivity in the near-well zone is a piecewise constant function whose plot consists of two steps. In the other two cases of filtrate invasion, some simplifying assumptions allow the resistivity curve to be plotted as three steps, one of which corresponds to the low resistivity of the annulus. Since the dynamic viscosity of the gas is lower than the viscosities of the liquid fluids, the filtrate invasion into gas formations is close to a piston-like displacement. As noted in [2], in this case of invasion, the annulus resulting from fast salt exchange between the water solutions is rather narrow, and, as a rule, is not detected by HFIIL devices: numerous data interpretations have shown that the dimensionless parameter σ characterizing the focusing properties of the sondes is approximately 0.7. Below, we consider the more complex

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case of mud filtrate invasion into to a formation which is originally saturated with three immiscible phases: oil, gas, and connate native salt water.

1. Mathematical Model of Filtrate Invasion into a Formation. In hole drilling, the effect of capillary forces is less significant than that of hydrodynamic forces. Under these conditions, the axisymmetric three-phase immiscible filtration in the near-well zone can be described by the Buckley–Leverett system [3]

$$\frac{1}{r}\frac{\partial}{\partial r}(rv_i) + m\,\frac{\partial s_i}{\partial t} = 0, \qquad v_i = -k_i f_i\,\frac{\partial h}{\partial r}, \tag{2}$$

where r is the coordinate, t is time, and m is the porosity, and $i = 0$, 1, and 2 (the subscript 2 refers to oil and the subscripts 0 and 1 refer to the gas and water phases, respectively). The first three equations of system (2) are the mass conservation laws for the moving phases and the next three are the generalized Darcy laws linking the radial velocities of motion v_i to the head gradient h, which is identical for all phases. The filtration coefficients k_i are inversely proportional to the dynamic viscosities μ_i of the phases and proportional to the formation permeability, and the relative phase permeabilities f_i are usually expressed as power functions s_i^n ($n \approx 3.5$) of the effective saturations. Since $\sum s_i = 1$, system (2) has the first integral

$$r(v_2 + v_0 + v_1) = r_w V(t), \tag{3}$$

where r_w is the borehole radius and $V(t)$ is the total volume velocity of the phases.

Introducing the generalized Leverett functions

$$\frac{F_i(s_2, s_0) = \alpha_i f_i(s_i)}{\sum\limits_i \alpha_i f_i(s_i)} \qquad \Big(\sum_i F_i = 1,\ \alpha_i = \mu_i/\mu_1,\ \alpha_1 = 1,\ \alpha_2 \leqslant 1,\ \alpha_0 \gg 1\Big),$$

from the laws of phase motion and formula (3) we obtain the following expressions for the phase filtration velocities:

$$rv_i = F_i(s_2, s_0) r_w V(t). \tag{4}$$

System (2) is simplified to

$$\frac{\partial s_0}{\partial \tau} + \frac{\partial F_0}{\partial s_2}\frac{\partial s_2}{\partial x} + \frac{\partial F_0}{\partial s_0}\frac{\partial s_0}{\partial x} = 0, \qquad \frac{\partial s_2}{\partial \tau} + \frac{\partial F_2}{\partial s_2}\frac{\partial s_2}{\partial x} + \frac{\partial F_2}{\partial s_0}\frac{\partial s_0}{\partial x} = 0, \tag{5}$$

where

$$\tau = \frac{2}{mr_w}\int\limits_0^t V(t)\,dt, \quad x = (r/r_w)^2$$

are new independent variables. The variable τ is linked to the standard radius r_n of volume filtrate invasion into the formation by the relation $r_n = r_w\sqrt{1+\tau}$. Let s_2^0, s_1^0, and $s_0^0 = 1 - (s_2^0 + s_1^0)$ be the initial oil, water, and gas saturations of the formation, respectively. The mobility of each phase depends largely on the product $\alpha_i f_i$. Other conditions being equal, the gas phase has the highest mobility ($\alpha_0 \approx 50$), and, hence, its displacement by fluids is close to piston flow. At the water–oil or water–gas ($i = 0$) displacement fronts $r = r_{f_i}$, the kinematic conditions

$$v_i(r_{f_i} - 0, t) - v_i(r_{f_i} + 0, t) = m[s_i(r_{f_i} - 0) - s_i(r_{f_i} + 0)]\,\frac{\partial r_{f_i}}{\partial t} \tag{6}$$

implied by the integral laws of mass conservation should be satisfied [3].

Depending on the relation between the oil (s_2^0) and gas (s_0^0) saturations of the formation, two cases are possible: 1) at all times, the gas displacement front is ahead of the oil displacement front ($r_{f_0} > r_f$); 2) the gas saturation of the formation is so small that the gas displacement front lags behind the oil displacement front ($r_{f_0} < r_f$). In the first case, from Eq. (6) and relation (4) for $i = 0$ with the initial condition $r_{f_0} = r_w$, we obtain

$$r_{f_0} = r_w\sqrt{1 + \tau F_0(s_2^0, s_0^0)/s_0^0}. \tag{7}$$

In the case of piston-like displacement, the gas saturation distribution in the near-well zone of the formation has the form of a piecewise constant function: $s_0 \equiv 0$ for $r_w < r < r_{f_0}(\tau)$ and $s_0 \equiv s_0^0$ for $r > r_{f_0}(\tau)$ [r_{f_0} is defined by formula (7)]. This distribution corresponds the transfer equation

$$\frac{\partial s_0}{\partial \tau} + \frac{F(s_2^0, s_0^0)}{s_0^0}\,\frac{\partial s_0}{\partial x} = 0,$$

which is formally obtained from the first equation of system (5) for $\partial s_2/\partial x = \partial s_0/\partial x = 0$ and

$$\frac{\partial F_0}{\partial s_2} \approx \frac{\Delta F_0}{\Delta s_0} = \frac{[F_0(s_2^0, s_0^0) - F(s_2^0, 0)]}{s_0^0} = \frac{F_0(s_2^0, s_0^0)}{s_0^0}.$$

The second equation of system (5) becomes

$$\frac{\partial s_2}{\partial \tau} + \frac{\partial F_2}{\partial s_2}(s_2, 0)\,\frac{\partial s_2}{\partial x} = 0, \qquad 1 < x \leqslant x_f = \Big(\frac{r_f}{r_w}\Big)^2. \tag{8}$$

The function $F(s_2, 0)$ coincides with the conventional Leverett function for two-phase filtration, and the behavior of the solutions of Eq. (8) is the same as found previously in [1, 2]. In particular, the following properties are the most important. If the initial oil saturation of the formation s_2^0 is higher than the maximum points $s_2 = s_{\max}$, at which $\partial^2 F_2(s_2, 0)/\partial s_2^2 = 0$, then the solution $s_2 = s_2(x, \tau)$ is discontinuous and the saturation at the displacement front $s_f < s_2^0$ is a solution of the transcendental equation

$$s_f = s_2^0 + \frac{[F_2(s_f, 0) - F_2(s_2^0, 0)]}{\partial F_2(s_f, 0)/\partial s_2}, \tag{9}$$

which follows from the kinematic condition (6). For $s_2^0 \leqslant s_{\max}$, the solution $s_2 = s_2(x, \tau)$ is continuous and $s_f = s_2^0$. The position of the oil displacement front is calculated by the formula

$$r_f = r_w \sqrt{1 + \tau\,\frac{\partial F_2}{\partial s_2}(s_f, 0)}, \tag{10}$$

and the oil saturation averaged over the area of the displacement zone $\langle s_2 \rangle$ does not depend on time and is defined as

$$\langle s_2 \rangle = s_f - \frac{F_2(s_f, 0)}{\partial F_2(s_f, 0)/\partial s_2}. \tag{11}$$

Let us consider the second case (the case of small s_0^0) with $r_{f_0} < r_f$. According to the piston-like displacement scheme, we have $s_0 = 0$ for $r < r_{f_0}$ and $s_0 = s_0^0$ for $r_{f_0} < r < r_f$. For all r, except for the point $r = r_{f_0}$, we have $\partial s_0/\partial r = 0$. Therefore, in this case, too, the equation of the sought function $s_2(x, \tau)$ is similar to Eq. (8). The difference is that the generalized Leverett function $F_2(s_2, s_0)$ does not coincide with the function $F_2(s_2, 0)$ for all r. In the interval $r_{f_0} < r < r_f$, $F_2(s_2, s_0) = F_2(s_2, s_0^0)$. However, since s_0^0 is a small quantity relative to the initial oil saturation s_2^0, in calculations one can ignore this difference and use approximations of the form (9)–(11). The position of the gas displacement front is calculated with reasonable accuracy by the formula

$$r_{f_0} = r_w \sqrt{1 + \tau F_0(\langle s_2 \rangle, s_0^0)/s_0^0},$$

which is obtained from the kinematic condition (6) assuming that the oil saturation s can be replaced by the average integral quantity $\langle s_2 \rangle$ which does not depend on the position of the front r_f.

2. Effect of Filtrate Invasion on the Electrical Conductivity of the Near-Well Zone. The electrical conductivity of a stratum can depend on its mineral composition, the degree of saturation of the pore space with electrolytes, salt concentration (more accurately, ionic strength of electrolytes), temperature, and other factors. As follows from the Archi law [4], other things being equal, the electric resistivity of a stratum of given composition is in inverse proportion to the square of the saturation of its pore space with electrolytes. Invasion of a water mud filtrate with a certain salt content c_p into a formation containing oil, gas, and relatively immovable native salt water with an unknown salt content c_0 is accompanied by fast salt exchange between these solutions. For simplification, we assume that in the region $r \in (r_w, r_f)$, the function $s_2(x, \tau)$ can be replaced by the area-averaged oil saturation $\langle s_2 \rangle$, which, according to formula (11), does not depend on the position of the front r_f and is determined only by the initial value of s_2^0. A diagram of interaction of the solutions for the case $r_{f_0} > r_f$ is given in Fig. 1. Figure 1a gives the water phase distribution and the salt content in the near-well zone ignoring mass exchange, and Fig. 1b gives the same quantities for the case of infinitely fast exchange [2].

For a solution with a salt concentration c_p, the mass balance equation is written as

$$(r_f^2 - r_w^2)(s_2^0 - \langle s_2 \rangle) + (r_{f0}^2 - r_w^2)s_0^0 = (r_{0z}^2 - r_w^2)(1 - \langle s_2 \rangle).$$

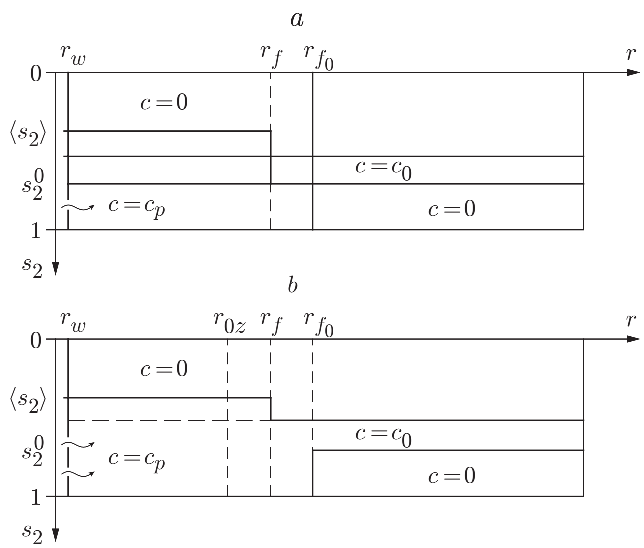

Fig. 1

From this equation, the position of the boundary ($r = r_{0z}$) separating these two solutions is given by the formula

$$r_{0z} = \sqrt{[s_0^0 r_{f_0}^2 + (s_2^0 - \langle s_2\rangle) r_f^2 + s_1^0 r_w^2]/(1 - \langle s_2\rangle)}. \tag{12}$$

The same result is obtained from the mass balance equation for salts dissolved in formation water with concentration c_0.

Taking into account the distribution of formation saturations with water solutions with concentrations c_p or c_0 and using the Archi law, we obtain four electric resistivity values for each of the zones shown in Fig. 1b:

$$R = \begin{cases} R_n = R_n^0/(1 - \langle s_2\rangle)^2, & r \in (r_w, r_{0z}), \\ R_{0z} = R_0[(1 - s_2^0 - s_0^0)/(1 - \langle s_2\rangle)]^2, & r \in (r_{0z}, r_f), \\ R_f = R_0[(1 - s_2^0 - s_0^0)/(1 - s_2^0)]^2, & r \in (r_f, r_{f0}), \\ R_0 = R^0/(1 - s_2^0 - s_0^0)^2, & r \in (r_{f0}, \infty). \end{cases} \tag{13}$$

Here R^0 is the electric resistivity of the formation with complete native water saturation and R_n^0 is the electric resistivity of the same formation with complete mud filtrate saturation.

As the initial gas content of the formation s_0^0 decreases, the gas displacement front r_{f_0} approaches the oil displacement front r_f. The length of the interval (r_f, r_{f_0}) tends to zero, and the effect of the resistivity R_f on the results of calculation of the induction resistivities $\bar{R}_i$ using formula (1) becomes insignificant. In the case of small s_0^0 ($r_{f_0} < r_f$), the mass balance equation for salts in a solution with a salt content c_p is written as

$$(r_f^2 - r_w^2)(s_2^0 - \langle s_2\rangle) + (r_{f0}^2 - r_w^2)s_0^0 = (r_{0z}^2 - r_w^2)(1 - \langle s_2\rangle) - (r_{0z}^2 - r_{f0}^2)s_0^0.$$

From this we obtain the following formula for the radius r_{0z} of the resistivity annulus:

$$r_{0z} = \sqrt{[(s_2^0 - \langle s_2\rangle) r_f^2 + s_1^0 r_w^2]/(1 - \langle s_2\rangle - s_0^0)}. \tag{14}$$

The electric resistivity distribution in the formation can be given by a piecewise constant function consisting of four steps:

$$R = \begin{cases} R_n = R_n^0/(1 - \langle s_2\rangle)^2, & r \in (r_w, r_{f0}), \\ R_{f0} = R_n[(1 - \langle s_2\rangle)/(1 - \langle s_2\rangle - s_0^0)]^2, & r \in (r_{f0}, r_{0z}), \\ R_{0z} = R_0[(1 - s_2^0 - s_0^0)/(1 - \langle s_2\rangle - s_0^0)]^2, & r \in (r_{0z}, r_f), \\ R_0 = R^0/(1 - s_2^0 - s_0^0)^2, & r \in (r_f, \infty). \end{cases} \tag{15}$$

For $s_0^0 = 0$, formulas (12) and (14) coincide with each other and with the expression for the radius of the annulus r_{0z} in oil formations obtained in [2]. Assuming that the centers r_i of sensitivities of the detector is a current coordinate,

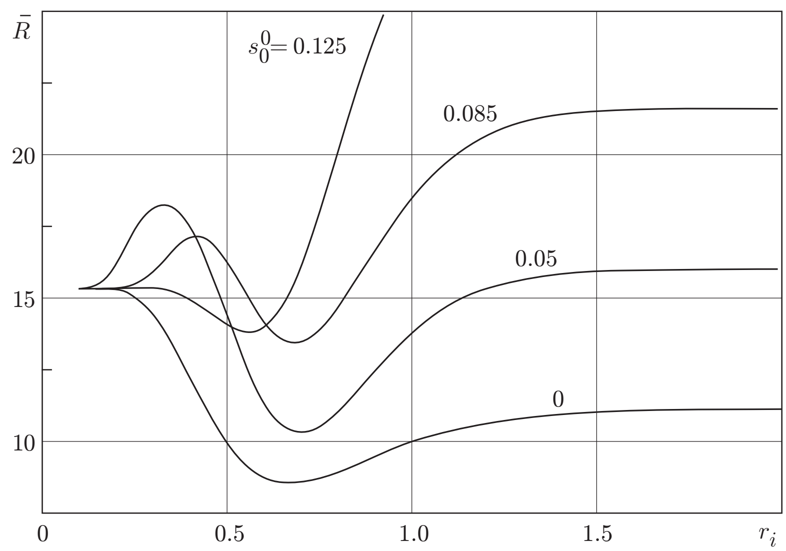

Fig. 2

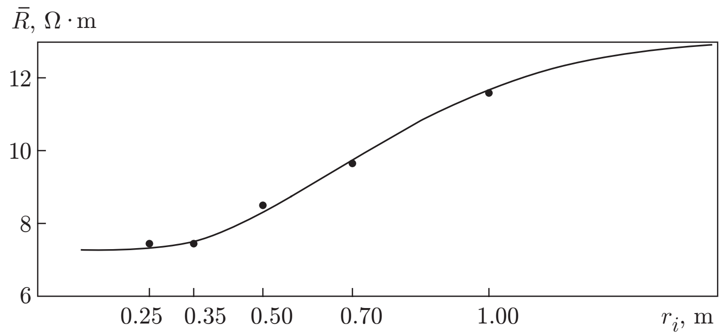

Fig. 3

from representation (1), we find the induction resistivities for any number of sondes with identical isoparameters [5]. According to the induction resistivities calculated by formulas (13) and (15), we obtain the following expressions:

$$\bar{R}(r_i) = \frac{R_n - R_{f0}}{2}\Big[1 + \operatorname{erf}\Big(\frac{1}{\sqrt{2}\sigma}\ln\frac{x_{f0}}{x_i} - \frac{\sigma}{\sqrt{2}}\Big)\Big] + \frac{R_{f0} - R_{0z}}{2}\Big[1 + \operatorname{erf}\Big(\frac{1}{\sqrt{2}\sigma}\ln\frac{x_{0z}}{x_i} - \frac{\sigma}{\sqrt{2}}\Big)\Big]$$

$$+ \frac{R_{0z} - R_0}{2}\Big[1 + \operatorname{erf}\Big(\frac{1}{\sqrt{2}\sigma}\ln\frac{x_f}{x_i} - \frac{\sigma}{\sqrt{2}}\Big)\Big] + R_0$$

for $r_{f_0} < r_f$ and

$$\bar{R}(r_i) = \frac{R_n - R_{0z}}{2}\Big[1 + \operatorname{erf}\Big(\frac{1}{\sqrt{2}\sigma}\ln\frac{x_{0z}}{x_i} - \frac{\sigma}{\sqrt{2}}\Big)\Big] + \frac{R_{0z} - R_f}{2}\Big[1 + \operatorname{erf}\Big(\frac{1}{\sqrt{2}\sigma}\ln\frac{x_f}{x_i} - \frac{\sigma}{\sqrt{2}}\Big)\Big]$$

$$+ \frac{R_f - R_0}{2}\Big[1 + \operatorname{erf}\Big(\frac{1}{\sqrt{2}\sigma}\ln\frac{x_{f0}}{x_i} - \frac{\sigma}{\sqrt{2}}\Big)\Big] + R_0$$

for $r_{f_0} > r_f$. Here ($x_i = (r_i/r_w)^2$, $x_f = (r_f/r_w)^2$, $x_{0z} = (r_{0z}/r_w)^2$, and $x_{f0} = (r_{f0}/r_w)^2$ $(i = 1, 2, \ldots)$.

Figure 2 gives plots of the function $\bar{R}(r_i)$ for the case $r_{f_0} < r_f$ for fixed values $s_2^0 = 0.7$, $r_n = 0.5$ m, $\alpha_2 = 0.16$, $\alpha_0 = 50$, $r_w = 0.1$ m, $R_n^0 = 4\ \Omega\cdot$ m, and $R^0 = 1\ \Omega\cdot$ m, and different values of s_0^0. The plots illustrate the effect of the gas content of the formation on the shape of the electromagnetic sounding curves.

As an example of practical application of the proposed approach, Fig. 3 gives interpretation of data from field measurements (shown by points on the plot) using a five-sonde HFIIL device. The hole was drilled in the Severoyur'evsk field of Surgutneftegaz. The data correspond to a depth of 2948.7 m. The following formation characteristics were obtained: oil content $s_2^0 = 0.843$, gas content $s_0^0 = 0.09$, radius of the oil displacement front $r_f = 1.02$ m, radius of the gas displacement front $r_{f0} = 0.7$ m, and radius of volume invasion $r_n = 0.74$ m. The standard deviation of the readings from the theoretical curve is 1.35%.

Conclusions. It is shown that as a result of drilling vertical boreholes in productive strata containing three immiscible phases: oil, gas, and a certain amount of mineralized native water, the electric resistance in the near-well zones can be written as a piecewise constant function of the radial coordinate. This function generally consists of four steps, the lower of which is detected by sondes as a resistivity annulus, i.e., a zone with decreased induction resistivity. Calculations show that even a small amount of gas in the formation has a significant effect on the electromagnetic log shape. In particular, for small s_0^0, the induction resistivity curves, have a local maximum in the neighborhood of the centers of sensitivities of the first two sondes in HFIIL devices, along with the minimum characteristic of oil formations.

This work was supported by the Russian Foundation for Basic Research (Grant No. 03-05-64210).

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